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Author(s): Cary Karp

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CALCULATING ATMOSPHERIC HUMIDITY

Cary Karp

Abstract—Derivations are given for the functional relationships between psychrometric and barometric observations, and the commonly encountered atmospheric humidity parameters. The use of a programmable pocket calculator is recommended for the practical application of the material.

1 Introduction

Monitoring atmospheric humidity is an important aspect of museum climate control. One of the most accurate and widely used humidity-measuring devices is the ventilated psychrometer (wet and dry bulb thermometers in various configurations). The different expressions of atmospheric humidity can all be calculated as functions of wet and dry bulb temperatures and barometric pressure. The most common method for reducing these data to humidity values is by reference to diagrams, tables or similar devices which are given for specific atmospheric pressures. This is quick and convenient but can be inaccurate if the actual barometric pressure differs from that for which the reference material is intended.

Although often of insignificant proportion, and therefore largely ignored in the literature, this inaccuracy can cause problems in several situations: when a psychrometer is used to adjust or calibrate humidity control equipment during extreme barometric conditions; when monitoring humidity during airplane transport (see Appendix); when dealing with high altitude locations [1]. If a barometer reading deviates more than 5% from the value for which an available psychrometric reference chart is calculated, especially at low temperatures and pressures, the chart may be unsuitable for critical use. Since atmospheric pressure can vary by more than 5% from the normal value this can unwittingly cause difficulty unless the museum climate is monitored with an accurate barometer.

Standard psychrometric charts are quite accurate and are completely adequate for routine climate monitoring in other than high-altitude conditions. If humidity must be known with a higher degree of accuracy than available reference material will permit, it can be calculated directly. A scientific pocket calculator is quite adequate for the necessary numerical manipulation, although its use might prove tedious. Programmable pocket calculators do not have this disadvantage and can therefore be recommended as an alternative to psychrometric charts for both critical and general work (in addition to being of great use in other applications within the field of conservation). This article is intended to aid in the implementation of this approach by providing a review of the mathematical basis for the calculation of atmospheric humidity.

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2 Psychrometric theory

A barometer measures the total pressure exerted on the surface of the earth by the mass of the atmosphere. Each gas in the air, including water vapor, has its own partial pressure. According to Dalton's Law, barometric pressure (p) can be regarded as the sum of the partial pressure of dry air (p_d) and the partial pressure of water vapor (e):

$$p = p_d + e \tag{1}$$

For a given temperature the partial pressure of water vapor cannot exceed a certain maximum value: the saturation partial pressure of water vapor (e_m). This maximum value is entirely dependent on temperature. The saturation partial pressure of water vapor is given in published tables. A modified Antoine Equation gives good approximations of tabular values in kilopascals for temperatures between −20°C and 50°C (1kPa = 10mbar = 7·5mmHg):

$$e_m = 0{\cdot}6105 \exp \frac{at}{b + t} \tag{2}$$

where t = temperature in °C
$a = 17{\cdot}25, b = 236{\cdot}9$ for water vapor over water
$a = 22{\cdot}27, b = 270{\cdot}8$ for water vapor over ice.

Below 0°C a solid surface draws water vapor from the air more readily than does a surface of water. The saturation partial pressure of water vapor is therefore lower over ice than over water. This affects psychrometric readings as the water on the wet bulb can freeze and the surface of the dry bulb can be below 0°C.

The actual partial pressure of water vapor (e) can be expressed in a simplified but entirely adequate 'psychrometer equation':

$$e = e_{mw} - Ap(t - t_w) \tag{3}$$

where e_{mw} = saturation partial pressure of water vapor at t_w (found by solving equation 2 for t_w)
A = a constant (explained below)
p = barometric pressure in kPa
t = dry bulb temperature in °C
t_w = wet bulb temperature in °C.

The psychrometer constant (A) will vary with the air speed past the wet bulb. At speeds greater than 3m/s this dependence can, however, be ignored. In this case,

$$A = \frac{c_{pd} + c_{pw}x}{(M_w/M_d)r_w} = 0{\cdot}00066 \quad (\text{if } x = 0{\cdot}0085) \tag{4}$$

where c_{pd} = specific heat capacity at constant pressure of dry air = 1·004kJ/kgK
c_{pw} = specific heat capacity at constant pressure of water vapor = 1·86kJ/kgK
x = the mixing ratio (see equation 12 below)
M_w = relative molecular mass of water vapor = 18·02
M_d = relative molecular mass of dry air = 28·96
r_w = specific latent heat of evaporation of water ≈ 2500kJ/kg

Since this constant is valid only for one value of x (equivalent to t = 20° and t_w = 15°) a correction factor is necessary for other conditions. This factor is very small and together with the temperature dependence of r_w also can be ignored. If the wet bulb is covered with ice r_w will be increased by the specific latent heat of fusion of ice (335kJ/kg), giving for sub-zero temperatures:

$$A = 0{\cdot}00058 \tag{4a}$$

The general equation of state for an ideal gas states that

$$p_i = \frac{(m_i/M_i)RT}{V} \tag{5}$$

where p_i = partial pressure of gas i
V = total volume
m_i = mass of gas i
M_i = relative molecular mass of gas i
R = the universal gas constant = 8·314J/mol
T = absolute temperature ($T = t + 273{\cdot}15$)

The density of dry air (ρ_d) in kg/m³ is

$$\rho_d = m_d/V \tag{6}$$

Substituting this into equation 5 gives:

$$\rho_d = \frac{p_d M_d}{RT} \tag{7}$$

From equation 1

$$p_d = p - e$$

Substituting this and the numerical constants into equation 7 gives

$$\rho_d = \frac{p - e}{0{\cdot}2871(t + 273{\cdot}15)} \tag{8}$$

Similarly, the density of water vapor (ρ_v) is

$$\rho_v = \frac{eM_w}{RT} \tag{9}$$

giving

$$\rho_v = \frac{e}{0{\cdot}4614(t + 273{\cdot}15)} \tag{10}$$

3 Definitions of various atmospheric humidity parameters

The various quantities used to express the moisture content of the atmosphere can now be defined. Absolute humidity is simply the density of atmospheric water vapor as given by equation 10. It states, in kg, the amount of water vapor present in one cubic meter of moist air. Since most rooms are described by their volume, absolute humidity is very useful in quantifying the amount of water vapor in a given room. This information is necessary when planning a humidification system (quantity of water needed, rate at which it must be dispersed into the air, etc.). Relative humidity (RH when expressed in percent, φ when expressed as a decimal fraction) can be seen as the ratio of absolute humidity to the maximum absolute humidity possible at a specified temperature. It is more correctly defined as the ratio of the partial pressure of water vapor to the saturation partial pressure of water vapor at a specified temperature:

$$\varphi = \frac{e}{e_m} \tag{11}$$

Relative humidity can thus be calculated by solving for e with equation 3, solving for e_m at the dry bulb temperature with equation 2, dividing the results, and multiplying by 100. Relative humidity is what is most often understood when the term 'humidity' stands alone. It is an expression of how readily air will absorb moisture and helps describe the way in which the atmosphere can affect hygroscopic and moisture-sensitive materials (low RH may cause wood to dry out and crack; high RH may cause glue to soften, mold to grow, metal to corrode, etc.).

Psychrometric charts do not usually provide accurate values for absolute humidity. The expression of the absolute moisture content of the air most commonly used is the mixing ratio (x). This is the ratio of absolute humidity to the density of dry air. Taking this ratio from equation 8 and 10 gives

$$x = \frac{0{\cdot}622e}{p - e} \qquad (12)$$

Substituting for e with equation 11 gives

$$x = \frac{0{\cdot}622\ \varphi e_m}{p - \varphi e_m} \qquad (13)$$

The mixing ratio can thus easily be calculated using psychrometric and barometric data. Although it does not have a physical dimension the mixing ratio can be expressed as the amount of wâter vapor in kg contained in one kg of dry air. This quantity serves the same practical purpose as absolute humidity, to which it is easily converted through multiplication by the density of dry air:

$$\varphi_v = x\varphi_d \qquad (14)$$

A further quantity which may be encountered is specific humidity. This is the ratio of absolute humidity to the density of moist air. Specific humidity does not differ from the mixing ratio by an amount large enough to warrant further treatment.

The dew point (t_d) is the temperature to which unsaturated air must be lowered at constant pressure to give 100% RH., i.e. when $e = e_m$. The dew point can thus easily be found by using equations 2 and 13 (see equations 2a and 13a below). Since water will condense on surfaces with sub-t_d temperatures, t_d is an important factor in determining safe minimum indoor air temperatures.

Although not a measure of humidity, specific enthalpy (h) in kJ/kg is often given in psychrometric charts. This is the total heat content of the moist air per unit mass dry air. Knowledge of specific enthalpy allows the energy requirements of a humidification system to be studied. Using the numerical constants given in equation 4 the specific enthalpy of one kg dry air and x kg water vapor is

$$h = 1{\cdot}004t + x(2500 + 1{\cdot}86t) \qquad (15)$$

4 Practical application

The material presented above can be used to calculate all values normally obtained by reference to psychrometric charts and tables. The least powerful of the programmable pocket calculators currently on the market can be programmed to provide values for the saturation partial pressures of water vapor (using equation 2). These values can then be used manually with equations 3 and 11 to calculate RH. Slightly more advanced calculators can do this without any non-programmed computation. The most powerful calculators are more than adequate to provide all the information discussed above. Individual workers will have practical use for varying amounts of this material. As an aid to individualized programming the relevant equations are repeated below.

4.1 List of symbols used

t = dry bulb temperature in °C
t_w = wet bulb temperature in °C
p = barometric pressure in kPa (1kPa = 10mbar)
e_m = saturation partial pressure of water vapor at dry bulb temperature in kPa
e_{mw} = saturation partial pressure of water vapor at wet bulb temperature in kPa
φ = relative humidity as a decimal fraction (RH/100)
A = the psychrometer constant: 0·00066 for water on wet bulb; 0·00058 for ice on wet bulb
a, b = coefficients used in finding saturation partial pressures of water vapor:
$a = 17{\cdot}25$, $b = 236{\cdot}9$ for 0°C and above;
$a = 22{\cdot}27$, $b = 270{\cdot}8$ for below 0°C
x = the mixing ratio (in kg/kg)
h = specific enthalpy in kJ/kg

4.2 Equations rewritten to isolate necessary variables

Any one of the following four variables can be found as a function of the other three: relative humidity, dry bulb temperature, wet bulb temperature, and barometric pressure. To solve for relative humidity use equations 2, 3, and 11. To solve for wet bulb temperature equations 3 and 11 are combined and written to give

$$\mathrm{f}(t_w) = \varphi e_m + Apt - e_{mw} - Apt_w = 0 \qquad (16)$$

t_w is a root of this equation and can be found iteratively by Newton's method:

$$t_{w_{n+1}} = t_{w_n} - \frac{\mathrm{f}(t_{w_n})}{\mathrm{f}'(t_{w_n})} \qquad (17)$$

where

$$\mathrm{f}'(t_w) = -Ap - \frac{e_{mw}ab\ \ln 10}{(b + t_w)^2} \qquad (18)$$

To solve for dry bulb temperature iterate as in the preceding case using

$$\mathrm{f}(t) = e_{mw} + Apt_w - \varphi e_m - Apt = 0 \qquad (19)$$

and

$$\mathrm{f}'(t) = -Ap - \frac{\varphi e_m ab\ \ln 10}{(b + t)^2} \qquad (20)$$

It should never be necessary to solve for barometric pressure.

Any one of the following four variables can be found as a function of the other three: the mixing ratio, dry bulb temperature, relative humidity, and barometric pressure. To solve for the mixing ratio use equation 13.

To solve for dry bulb temperature equation 13 is written to solve for e_m

$$e_m = \frac{xp}{\varphi(0{\cdot}622 + x)} \qquad (13a)$$

which is then used in equation 2 written to solve for t:

$$t = \frac{b \ln(e_m/0{\cdot}6105)}{a - \ln(e_m/0{\cdot}6105)} \qquad (2a)$$

To solve for relative humidity equation 13 is written to solve for φ (and the result multiplied by 100 to give RH):

$$\varphi = \frac{xp}{e_m(0{\cdot}622 + x)} \qquad (13b)$$

It should never be necessary to solve for barometric pressure.

Any one of the following three variables can be found as a function of the other two: specific enthalpy, the mixing ratio, and dry bulb temperature.

To solve for specific enthalpy use equation 15.

To solve for the mixing ratio equation 15 is rewritten for x:

$$x = \frac{h - 1{\cdot}004t}{2500 + 1{\cdot}86t} \qquad (15a)$$

To solve for dry bulb temperature equation 15 is rewritten for t:

$$t = \frac{h - 2500x}{1{\cdot}004 + 1{\cdot}86x} \qquad (15b)$$

5 Conclusion

Conventional methods for the evaluation of psychrometric data using charts or tables are entirely satisfactory for most routine museum humidity control. Situations may nonetheless arise in which necessary parameters require direct calculation from both psychrometric and barometric data. Programmable pocket calculators can be used to good advantage in this regard and are convenient for routine application, as well. If extensive psychrometric measurements are made in a wide variety of situations it is, therefore, suggested that a barometer and a programmable calculator be added to the usual complement of climate measuring devices.

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Appendix: The air transport environment

Museum objects are often transported by air and thereby exposed to what one could expect to be abnormal climatic conditions. Despite this there is extremely little literature of use in determining the actual risks which might be encountered [2, 3] and which would help in planning suitable precautionary measures.

To obtain necessary data (and to test the practical accuracy of the material presented in this article) direct measurements were made of various climatic parameters in the passenger cabins and baggage holds on 16 flights with the following aircraft types: DC-8, DC-9, DC-10, B727. Baggage-hold temperatures were monitored with a recording bimetal thermometer. Passenger cabin temperatures were measured with wet and dry bulb thermometers, cabin pressures with an aneroid barometer, and cabin relative humidities both with a hair hygrometer and by reduction from the barometric and psychrometric data. No great differences were noticed between the data obtained either on the various aircraft types or on individual flights.

What were essentially minimum cabin pressures were noted within the first 45 minutes of every flight, fluctuating little around these levels until starting descent. The lowest pressure recorded was 763mbar.* Typical values ranged between 820–850mbar over longer periods of time, suggesting typical jet aircraft cabin pressures to be between 20–25% lower than sea-level normal. Cabin temperatures were consistently held between 20–25°C. Baggage-hold temperatures levelled off at 5–10°C although they were subject to short-term occasional disruption by as much as ±5°C. Minimum stable values were attained after several hours in flight. Initial conditions were similar to those prevailing outdoors before the start of flight. Relative humidity readings made with the hair hygrometer and those calculated from the barometric and psychrometric data were in consistent close agreement, suggesting the accuracy of the equations used for the calculation (although one might expect the performance of both measuring devices to be less than optimal under cabin conditions).

Cabin relative humidities dropped steadily after take-off, stabilizing at minimum values ranging between 10–20% some time after reaching maximum altitude and remaining at these levels until descent. Both these minima and the time elapsed until they were attained appear to correlate to passenger density. Cabin temperatures can be up to 70°C higher than those outside the airplane and one would expect unpopulated cabin minimum relative humidities considerably lower than those observed.

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*Despite the stringent use of SI units in the body of the text of this article, the millibar remains the more common unit in meteorological practice.

The water vapor produced by the passengers would appear to contribute appreciably to the measured relative humidity, as might be expected.* Baggage-hold RH should be higher than in the cabin due to its lower temperature. Considering the passengers as a source of atmospheric water vapor, however, conditions in the hold may not vary from those in the cabin solely as a function of temperature.

It must be noted that conditions on cargo flights, i.e. with no passenger cabin, differ significantly from those described above. The subject demands further study.

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*A sedentary adult normally evaporates somewhat more than 40g of water per hour, the low RH and pressure in the cabin increasing this figure by 20%. Therefore one adult passenger can maintain 3m^3 of otherwise totally dry cabin space at 15% RH if the air in this space is completely changed once every three minutes.

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CARY KARP, BA, City University of New York, 1969; doctoral dissertation in progress at Uppsala University. Since 1971, director of the conservation facility at the Music Museum, Stockholm. Since 1973, curator of the musical instrument collections of the same museum. *Author's address: SMS-Musikmuseet, Box 16326, S–10326 Stockholm, Sweden.*

Résumé—On donne les relations dérivées entre les observations psychrométriques et barométriques, en fonction des variations du taux d'humidité. On recommande l'utilisation d'une calculatrice de poche programmable pour l'exploitation rationnelle de ces données.

Auszug—Es werden Ableiteungen gegeben für die funktionellen Beziehungen zwischen psychrometrischen und barometrischen Beobachtungen und den allgemein anzutreffenden Luftfeuchtigkeitsparametern. Die Anwendung eines programmierbaren Taschenrechners wird für die praktische Anwendung des Materials empfohlen.